Solving Ordinary Differential Equations with Variable Coefficients Using the New Bayawa Transform

Hisham Zawam Rashdi,
Department of Mathematics,

Faculty of Arts and Sciences, Kasr Khiar.

Elmergib University - Libya.

hzrashdi@elmergib.edu.ly

ملخص:

في هذه الورقة، تم تطبيق تحويل تكاملي جديد لإيجاد حل المعادلات التفاضلية العادية ذات المعاملات المتغيرة. المعادلات التفاضلية العادية لها أهمية كبيرة في مجالات علمية مختلف بشكل عام والرياضيات الفيزيائية والتطبيقية بشكل خاص.

التحويل التكاملي الجديد يسمى تحويل باياوا. يعد تحويل باياوا أحد التحويلات الحديثة التي تم تطويرها في الاشهر الاخيرة. يمكن لهذا التحويل أن يحل المعادلات التفاضلية العادية ذات المعاملات الثابتة بشكل عام، وأيضا يمكن استخدامه لحل المعادلات التفاضلية العادية ذات المعاملات المتغيرة، وهو ما تم بحثه في هذه الورقة.

في هذا العمل، قمنا بتنفيذ عدة طرق واستنتاجات لاستخلاص عدد من الصيغ والنتائج الهامة. تم بنجاح استخدام هذه النتائج لحل المعادلات التفاضلية العادية ذات المعاملات المتغيرة باستخدام تحويل باياوا ومعكوسه. الحلول الناتجة أعطتنا فكرة واضحة بأن استخدام مثل هذه الاساليب المبتكرة في حل هذا النوع من المعادلات يمثل مساراً واعداً للتطورات المستقبلية.

ABSTRACT:

A new integral transform was applied in this paper to find the solution of ordinary differential equations (ODEs) with variable coefficients. ODEs are of great importance in various scientific fields, especially physical and applied mathematics.

The new integral transform is called Bayawa transform. Bayawa transform is one of the modern transformations that have been developed in recent months. The Bayawa transform can be solved ODEs with constant coefficients in general. Furthermore, it can be used to solve ODEs with variable coefficients, which was discussed by this paper.

Keywords: Bayawa Transform, Inverse Bayawa Transform, Ordinary Differential Equations with Variable Coefficients.

1) Introduction

Ordinary Differential Equations (ODEs) are crucial in describing various physical, engineering and economic systems. Among these ODEs is ordinary differential equation with variable coefficients which are particularly challenging due to their complexity and the intricate behavior of the dependent variable influenced by coefficients that change with respect to the independent variable.

Usually solving this type of equations is complicated by using normal methods such as the series method. On the other hand, integral transforms play a big role in solving such equations [1]. One of the most important advantages of integral transformations is obtaining efficient and accurate solutions to differential equations without complicated calculations, as indicated by [4] & [5]. Furthermore, applied mathematics, theoretical mechanics, statistics and mathematical physics problems have become dependent on integral transforms as an important tool to find solution of those problems [3] & [5].

Bayawa transform is one of these modern transforms which has been developed recently. “Bayawa Integral transform has been introduced to facilitate the process of solving ordinary and partial differential equations in time domain; it has been derived from the classical Fourier integral" [2].

The definition of Bayawa transform as mentioned by Zayyanu and Haliru in [2] of a piecewise continuous exponential order function $f(t); t \geq 0$ is defined as the following integral equation:

$$\mathfrak{B}\{f(t)\} = v^2 \int_0^\infty f(t)\, e^{-\frac{t}{v^2}}\, dt = \mathrm{Z}(v), h_1 \leq v \leq h_2 : h_1 \,\&\, h_2 > 0, \quad (1)$$

where v is real parameter and $\mathfrak{B}$ is the Bayawa transform operator, and h_1 & h_2 may be finite or infinite. Therefore, purpose of this paper will be to apply this transform to solve some problems of ordinary differential equations with variable coefficients and show its efficiency.

2) Bayawa transform and Inverse Bayawa transform of Some Functions:

The following formulas for the Bayawa transform of some functions have been written and summarized in table (1) as are from their source [2].

No.	$f(t)$	$\mathfrak{B}\{f(t)\} = Z(v)$	$Z(v)$	$f(t) = \mathfrak{B}^{-1}\{Z(v)\}$
1	1	v^4	v^4	1
2	t	v^6	v^6	t
3	t^2	$2!\,v^8$	v^8	$\frac{1}{2!}t^2$
4	t^3	$3!\,v^{10}$	v^{10}	$\frac{1}{3!}t^3$
5	$t^n; n \in \mathbb{N}$	$n!\,v^{2n+4}$	$v^{2n+4}; n \in \mathbb{N}$	$\frac{1}{n!}t^n$
6	e^{at}	$\frac{v^4}{1-av^2}$	$\frac{v^4}{1-av^2}$	e^{at}
7	$\sin at$	$\frac{av^6}{1+a^2v^4}$	$\frac{av^6}{1+a^2v^4}$	$\sin at$
8	$\cos at$	$\frac{v^4}{1+a^2v^4}$	$\frac{v^4}{1+a^2v^4}$	$\cos at$
9	$\sinh at$	$\frac{av^6}{1-a^2v^4}$	$\frac{av^6}{1-a^2v^4}$	$\sinh at$
10	$\cosh at$	$\frac{v^4}{1-a^2v^4}$	$\frac{v^4}{1-a^2v^4}$	$\cosh at$

Table (1): Bayawa transform and Inverse Bayawa transform of some functions.

3) Bayawa Transform of Derivatives:

Let $Z(v)$ is Bayawa integral transform of $f(t)$ denoted by $\mathfrak{B}\{f(t)\}$ that is $[\mathfrak{B}\{f(t)\} = Z(v)]$, then we will have the flowing formulas which have been proofed in [2]:

$\mathfrak{B}\ \{f'(t)\} = \frac{1}{v^2} Z(v) - v^2 f(0).$ **(2)**

$\mathfrak{B}\ \{f''(t)\} = \frac{1}{v^4} Z(v) - f(0) - v^2 f'(0).$ **(3)**

$\mathfrak{B}\{f^{(n)}(t)\} = \frac{1}{v^{2n}} Z(v) - \sum_{k=0}^{n-1} v^{-2n+2k+4} f^{(k)}(0).$ **(4)**

4) Bayawa Transform of Functions $tf(t)$ & $t^2f(t)$:

If $\mathfrak{B}\{f(t)\} = Z(v)$, then:

i. $\mathfrak{B}\{tf(t)\} = \frac{v^3}{2}\frac{d}{dv} Z(v) - v^2 Z(v)$.

Proof. Since, $\mathfrak{B}\{f(t)\} = v^2 \int_0^\infty f(t)\, e^{-\frac{t}{v^2}}\, dt = Z(v)$

$\therefore \quad \frac{d}{dv} Z(v) = \frac{d}{dv}\left(v^2 \int_0^\infty f(t)\, e^{-\frac{t}{v^2}}\, dt\right)$

$= v^2 \int_0^\infty \frac{2t}{v^3} f(t) e^{-\frac{t}{v^2}} dt + 2v \int_0^\infty f(t) e^{-\frac{t}{v^2}} dt$

$\Rightarrow \quad \frac{v^3}{2}\frac{d}{dv} Z(v) = \mathfrak{B}\{tf(t)\} + v^2 Z(v)$

$\Rightarrow \quad \mathfrak{B}\{tf(t)\} = \frac{v^3}{2}\frac{d}{dv} Z(v) - v^2 Z(v).$ **(5)**

ii. $\mathfrak{B}\{t^2 f(t)\} = \frac{v^6}{4}\frac{d^2}{dv^2} Z(v) - \frac{v^5}{4}\frac{d}{dv} Z(v)$.

Proof. Since, $\mathfrak{B}\{tf(t)\} = \frac{v^3}{2}\frac{d}{dv} Z(v) - v^2 Z(v)$

$= \frac{v^3}{2}\frac{d}{dv}\mathfrak{B}\{f(t)\} - v^2 \mathfrak{B}\{f(t)\}.$

$\therefore \quad \mathfrak{B}\{t^2 f(t)\} = \frac{v^3}{2}\frac{d}{dv}\mathfrak{B}\{tf(t)\} - v^2\mathfrak{B}\{tf(t)\}$

$= \frac{v^3}{2}\frac{d}{dv}\left(\frac{v^3}{2}\frac{d}{dv} Z(v) - v^2 Z(v)\right) - v^2\left(\frac{v^3}{2}\frac{d}{dv} Z(v) - v^2 Z(v)\right)$

$\Rightarrow \quad \mathfrak{B}\{t^2 f(t)\} = \frac{v^6}{4}\frac{d^2}{dv^2} Z(v) - \frac{v^5}{4}\frac{d}{dv} Z(v).$ **(6)**

5) Bayawa Transform of Functions $tf'(t)$ & $t^2f'(t)$:

i. $\mathfrak{B}\{tf'(t)\} = \frac{v}{2}\frac{d}{dv} Z(v) - 2Z(v) - \frac{v^5}{2}\frac{d}{dv} f(0).$

Proof. In Eq. (**5**), we will put $f'(t)$ instead of $f(t)$ which give us:

$\mathfrak{B}\{tf'(t)\} = \frac{v^3}{2}\frac{d}{dv}\mathfrak{B}\{f'(t)\} - v^2\mathfrak{B}\{f'(t)\}.$

From Eq. (**2**) we have: $\mathfrak{B}\{f'(t)\} = \frac{1}{v^2}Z(v) - v^2 f(0)$, therefore:

$$\mathfrak{B}\{tf'(t)\} = \frac{v^3}{2}\frac{d}{dv}\left(\frac{1}{v^2}Z(v) - v^2 f(0)\right) - v^2\left(\frac{1}{v^2}Z(v) - v^2 f(0)\right)$$

$$\Rightarrow \quad \mathfrak{B}\{tf'(t)\} = \frac{v}{2}\frac{d}{dv}Z(v) - 2Z(v) - \frac{v^5}{2}\frac{d}{dv}f(0). \qquad (7)$$

ii. $\mathfrak{B}\{t^2 f'(t)\} = \frac{v^4}{4}\frac{d^2}{dv^2}Z(v) - \frac{7v^3}{4}\frac{d}{dv}Z(v) + 2v^2 Z(v) + \frac{v^4}{2}\frac{d}{dv}f(0) - \frac{v^5}{4}\frac{d^2}{dv^2}f(0).$

Proof. From Eq. (**5**), we will obtain:

$$\mathfrak{B}\{t^2 f'(t)\} = \frac{v^3}{2}\frac{d}{dv}\mathfrak{B}\{tf'(t)\} - v^2\mathfrak{B}\{tf'(t)\}$$

$$\Rightarrow \quad = \frac{v^3}{2}\frac{d}{dv}\left(\frac{v}{2}\frac{d}{dv}Z(v) - 2Z(v) - \frac{v^5}{2}\frac{d}{dv}f(0)\right) - v^2\left(\frac{v}{2}\frac{d}{dv}Z(v) - 2Z(v) - \frac{v^5}{2}\frac{d}{dv}f(0)\right),$$

which give us:

$$\mathfrak{B}\{t^2 f'(t)\} = \frac{v^4}{4}\frac{d^2}{dv^2}Z(v) - \frac{7v^3}{4}\frac{d}{dv}Z(v) + 2v^2 Z(v) + \frac{v^4}{2}\frac{d}{dv}f(0) - \frac{v^5}{4}\frac{d^2}{dv^2}f(0). \qquad (8)$$

In the same way as above, we can derive and conclude the following formulas:

$$\mathfrak{B}\{tf''(t)\} = \frac{1}{2v}\frac{d}{dv}Z(v) - \frac{3}{v^2}Z(v) + v^2 f(0) - \frac{v^3}{2}\frac{d}{dv}f(0) - \frac{v^5}{2}\frac{d}{dv}f'(0). \qquad (9)$$

$$\mathfrak{B}\{t^2 f''(t)\} = \frac{v^2}{4}\frac{d^2}{dv^2}Z(v) - \frac{9v}{4}\frac{d}{dv}Z(v) + 6Z(v) - v^7\frac{d}{dv}f'(0) - \frac{v^6}{4}\frac{d^2}{dv^2}f(0) - \frac{v^8}{4}\frac{d^2}{dv^2}f'(0). \qquad (10)$$

6) Theorem: If $\mathfrak{B}\{f(t)\} = Z(v)$, then: $\mathfrak{B}\{t^n e^{at}\} = \frac{n!(v^{2n+4})}{(1-av^2)^{n+1}}$.

Proof. From Eq. (**1**), we get:

$$\mathfrak{B}\{t^n e^{at}\} = v^2\int_0^\infty t^n e^{at} e^{-\frac{t}{v^2}}\,dt = v^2\int_0^\infty t^n e^{-\left(\frac{1}{v^2}-a\right)t}\,dt.$$

Let: $w = \left(\frac{1}{v^2} - a\right)t \Rightarrow t = \left(\frac{v^2}{1-av^2}\right)w \Rightarrow dt = \left(\frac{v^2}{1-av^2}\right)dw.$

$$\Rightarrow \quad v^2 \int_0^\infty t^n e^{-\left(\frac{1}{v^2}-a\right)t}\, dt \quad = v^2 \int_0^\infty \left(\frac{v^2}{1-av^2}\right)^n w^n e^{-w} \left(\frac{v^2}{1-av^2}\right) dw.$$

$$= \frac{v^{2n+4}}{(1-av^2)^{n+1}} \int_0^\infty w^n e^{-w}\, dw.$$

$$\because \ \Gamma(n) = \int_0^\infty z^{n-1} e^{-z}\, dz \quad \Rightarrow \Gamma(n+1) = \int_0^\infty z^n e^{-z}\, dz = n!\,.$$

$$\therefore \ \int_0^\infty w^n e^{-w}\, dw = n! \qquad \Rightarrow v^2 \int_0^\infty t^n e^{-\left(\frac{1}{v^2}-a\right)t}\, dt = \frac{n!\left(v^{2n+4}\right)}{(1-av^2)^{n+1}}$$

$$\Rightarrow \qquad \mathfrak{B}\{t^n e^{at}\} = \frac{n!\left(v^{2n+4}\right)}{(1-av^2)^{n+1}}\ . \qquad (11)$$

7) Applications:

In this section, we will apply the results obtained previously, to a set of problems and then use the inverse Bayawa transform to obtain the solutions.

Problem (1): Solve the differential equation:

$$y' - 2y = 0\,; \ \ y(0) = 0.$$

Solution: Taking Bayawa transform to both sides of given equation:

$$\mathfrak{B}\{ty'\} - 2\mathfrak{B}\{y(t)\} = 0.$$

$$\frac{v}{2}\frac{d}{dv} Z(v) - 2Z(v) - \frac{v^5}{2}\,\frac{d}{dv}(0) - 2Z(v) = 0$$

$$\Rightarrow \qquad \frac{d}{dv} Z(v) - \frac{8}{v} Z(v) = 0,$$

which is a linear differential equation with the integrative factor: $\lambda = \frac{1}{v^8}$. The solution will be in the form:

$$Z(v) = Cv^8 \qquad \Rightarrow \ y(t) = C\mathfrak{B}^{-1}\{v^8\} \ \Rightarrow \quad y(t) = \frac{Ct^2}{2}.$$

Problem (2): Solve the differential equation: $ty'' - ty' - y = 0,$

with the initial condition: $y(0) = 0 \ \ \& \ \ y'(0) = 2.$

Solution: $\mathfrak{B}\{ty''\} - \mathfrak{B}\{ty'\} - \mathfrak{B}\{y\} = 0.$ From formulas (7) & (9), we get:

$$\frac{1}{2v}\frac{d}{dv} Z(v) - \frac{3}{v^2} Z(v) + v^2(0) - \frac{v^3}{2}\frac{d}{dv}(0) - \frac{v^5}{2}\frac{d}{dv}(2)$$
$$- \left(\frac{v}{2}\frac{d}{dv} Z(v) - 2Z(v) - \frac{v^5}{2}\,\frac{d}{dv}(0)\right) - Z(v) = 0$$

$$\Rightarrow \left(\frac{1}{2v} - \frac{v}{2}\right)\frac{d}{dv}\, Z(v) - \left(\frac{3}{v^2} - 1\right) Z(v) = 0. \qquad \text{Let} \qquad Z(v) = S.$$

$$\Rightarrow \qquad (v - v^3)\frac{dS}{dv} - (6 - 2v^2)S = 0 \quad \Rightarrow \quad \frac{dS}{S} = \frac{6-2v^2}{v(1-v^2)}\, dv.$$

Take synthetic division method, we will have:

$$\frac{6-2v^2}{v(1-v^2)} = \frac{A}{v} + \frac{B}{1-v} + \frac{C}{1+v} \quad \Rightarrow \quad A = 6, \; B = 2 \;\&\; C = -2.$$

$$\frac{dS}{S} = \frac{6}{v}dv - \frac{2}{v-1}dv - \frac{2}{v+1}dv \Rightarrow \ln S = 6\ln v - 2\ln(v-1) - 2\ln(v+1) + \ln C.$$

$$\Rightarrow S = \frac{Cv^6}{(1-v^2)^2} \qquad \Rightarrow Z(v) = \frac{Cv^6}{(1-v^2)^2}.$$

Now, take the inverse Bayawa transform by using formula (**11**), we get:

$$y(t) = Cte^t.$$

$$\because\; y'(0) = 2 \Rightarrow C = 2 \qquad \Rightarrow y(t) = 2te^t.$$

Problem (3): Solve the differential equation: $ty'' - y' = 4t^2$, with the initial condition: $y(0) = 1 \;\&\; y'(0) = 0$.

Solution: Applying Bayawa transform to give us:

$$\mathfrak{B}\{ty''\} - \mathfrak{B}\{y'\} = \mathfrak{B}\{4t^2\}$$

$$\frac{1}{2v}\frac{d}{dv}Z(v) - \frac{3}{v^2}Z(v) + v^2(1) - \frac{v^3}{2}\frac{d}{dv}(1) - \frac{v^5}{2}\frac{d}{dv}(0) + \frac{1}{v^2}Z(v) - v^2(1) = 8v^8.$$

$$\Rightarrow \frac{d}{dv}Z(v) - \frac{4}{v}Z(v) = 16v^9$$

which is a linear differential equation and it has the integrative factor: $\lambda = \frac{1}{v^4}$.

$$\Rightarrow Z(v) = \frac{8}{3}v^{10} + Cv^4.$$

Now, taking the inverse Bayawa transform of the above equation yields:

$$y(t) = \frac{8}{3}\mathfrak{B}^{-1}\{v^{10}\} + C\mathfrak{B}^{-1}\{v^4\} \quad \Rightarrow \quad y(t) = \frac{4}{9}t^3 + C.$$

$$\because y(0) = 1 \quad \Rightarrow \quad C = 1. \quad \Rightarrow \quad y(t) = \frac{4}{9}t^3 + 1.$$

Problem (4): Solve the differential equation: $t^2y'' - ty' + y = 5$, with: $y(0) = 5 \;\&\; y'(0) = 3$.

Solution: Taking Bayawa transform of the given equation which gives:

$$\mathfrak{B}\{ty''\} - \mathfrak{B}\{ty'\} + \mathfrak{B}\{y\} = 5\mathfrak{B}\{1\}$$

$$\frac{v^2}{4}\frac{d^2}{dv^2}Z(v) - \frac{9v}{4}\frac{d}{dv}Z(v) + 6Z(v) - v^7\frac{d}{dv}(3) - \frac{v^6}{4}\frac{d^2}{dv^2}(5) - \frac{v^8}{4}\frac{d^2}{dv^2}(3) - \left(\frac{v}{2}\frac{d}{dv}Z(v) - 2Z(v) - \frac{v^5}{2}\frac{d}{dv}(5)\right) + Z(v) = 5v^4$$

$$\Rightarrow v^2\frac{d^2}{dv^2}Z(v)-11v\frac{d}{dv}Z(v)+36Z(v)=20v^4. \qquad (\boldsymbol{I})$$

Here, we will consider $w=Z(v)$, which leads the Cauchy-Euler differential equation, and also consider:

$$v=e^x \Rightarrow x=\ln v \Rightarrow \frac{d^2w}{dv^2}=e^{-2x}\left(\frac{d^2w}{dx^2}-\frac{dw}{dx}\right), \frac{dw}{dv}=e^{-x}\frac{dw}{dx}.$$

Substitutions of above work in $(\boldsymbol{I})$ gives us: $\frac{d^2w}{dx^2}-12\frac{dw}{dx}+36w=20e^{4x}$, which has the general solution: $w=C_1v^6+C_2v^6\ln v+5v^4$.

Note that if we require $y(0)$ being finite, we are forced to conclude that $C_2=0$. $\Rightarrow Z(v)=C_1v^6+5v^4$.

Appling the inverse Bayawa transform yields: $y(t)=C_1t+5$.

$\because y'(0)=3 \Rightarrow C_1=3 \Rightarrow y(t)=3t+5.$

Problem (5): Solve the differential equation:

$$ty''+(1-2t)y'-2y=0; \quad y(0)=1 \;\&\; y'(0)=2.$$

Solution: Applying Bayawa transform yields:

$$\mathfrak{B}\{ty''\}+\mathfrak{B}\{y'\}-2\mathfrak{B}\{ty'\}-2\mathfrak{B}\{y\}=0$$

$$\frac{1}{2v}\frac{d}{dv}Z(v)-\frac{3}{v^2}Z(v)+v^2(1)-\frac{v^3}{2}\frac{d}{dv}(1)-\frac{v^5}{2}\frac{d}{dv}(2)+\frac{1}{v^2}Z(v)-v^2(1)-2\left(\frac{v}{2}\frac{d}{dv}Z(v)-2Z(v)-\frac{v^5}{2}\frac{d}{dv}(1)\right)-2Z(v)=0$$

$$\Rightarrow \quad \left(\frac{1}{2v}-v\right)\frac{d}{dv}Z(v)-\left(\frac{2}{v^2}-2\right)Z(v)=0.$$

$$\Rightarrow \frac{du}{dv}-\frac{(4-4v^2)}{v(1-2v^2)}u=0 \qquad \Rightarrow \qquad \frac{du}{u}=\frac{(4-4v^2)}{v(1-2v^2)}dv \;:\; Z(v)=u.$$

Take synthetic division method, we will get:

$$\frac{(4-4v^2)}{v(1-2v^2)}=\frac{A}{v}+\frac{B}{1-\sqrt{2}v}+\frac{C}{1+\sqrt{2}v} \quad \Rightarrow \quad A=4,\; B=\sqrt{2} \;\&\; C=-\sqrt{2}.$$

$$\Rightarrow \quad \frac{du}{u}=\frac{6}{v}dv-\frac{\sqrt{2}}{\sqrt{2}v-1}dv-\frac{\sqrt{2}}{\sqrt{2}v+1}dv$$

$$\Rightarrow \quad \ln u=4\ln v-\ln\left(\sqrt{2}v-1\right)-\ln\left(\sqrt{2}v+1\right)+\ln C.$$

$$\Rightarrow \quad u=\frac{Cv^4}{1-2v^2} \qquad \Rightarrow \qquad Z(v)=\frac{Cv^4}{1-2v^2}.$$

From table $(\mathbf{1})$, we have: $y(t)=Ce^{2t}$.

$\because \quad y'(0)=2 \Rightarrow C=1 \quad \Rightarrow \quad y(t)=e^{2t}.$

8) Conclusion

The new Bayawa transform presents a novel and efficient approach to solving ordinary differential equations with variable coefficients. This was discussed in this paper and successfully applied to solve this type of ODE. We applied the Bayawa transform to a number of problems, and all obtained solutions satisfied the given equations. Further research into the applications and limitations of this transform holds promise for advancing our understanding and solving complex differential equations with variable coefficients.

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